

**DEVELOPMENT OF EXPONENTIAL MODEL TO FIND FATIGUE
CRACK GROWTH AND RESIDUAL LIFE FOR
CONSTANT AMPLITUDE LOADING**

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF

BACHELOR OF TECHNOLOGY

IN

MECHANICAL ENGINEERING

By:

TIKENDRA DEWANGAN (109ME0643)



DEPARTMENT OF MECHANICAL ENGINEERING

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Under the guidance of

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NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

CERTIFICATE

This is to certify that the thesis entitled “Development of Exponential Model to Find Fatigue Crack Growth and Residual Life for Constant Amplitude Loading” submitted by Tikendra Dewangan (109ME0643) in the partial fulfillment of the requirements for the award of Bachelor of Technology degree in Mechanical Engineering at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

Date:

Prof. P. K. Ray

Department of Mechanical Engineering

National Institute of Technology, Rourkela

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Place:

Tikendra Dewangan

Date:

109ME0643

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ABSTRACT

Fatigue results in the nucleation of cracks in the structure. Over a period of time, the sizes of these cracks reach a critical dimension and this result into a catastrophic failure of the structure. Hence it is necessary to replace the structure before the crack reaches the critical value. Thus evaluation of fatigue crack growth rate and fatigue life is necessary for safety as well as from economic point of view.

In this project, a modified exponential model has been developed to determine the fatigue crack growth rate and residual fatigue life of the structure. The modeling is based on the tests carried out on single edge notched tension (SENT) specimens in Instron 8502 machine, at constant amplitude loading conditions. To calculate the specific growth rate (m) in the exponential model, an expression has been derived using dimensional analysis. The values of different constants in the expression for specific crack growth rate have been obtained using the MATLAB software.

CHAPTER 1

INTRODUCTION

1.1 FATIGUE

As described in Wikipedia, Fatigue is the progressive and localized structural damage that occurs when a material is subjected to cyclic loading. The maximum stress values can be less than the ultimate tensile stress value, and may even be below the yield stress limit of the material[1]. Fatigue occurs when the material is exposed through repeated loading and unloading. When the loads are above a particular threshold value, microscopic cracks start to occur at the stress concentrators, for example the surface, persistent slip bands, and grain interfaces. Over a period of time the crack will reach a critical size, and then the structure will suddenly fracture. Usually the shape of structure has a significant effect on the fatigue life, for example square holes or sharp corners generally lead to increased local stresses where fatigue cracks can nucleate. Round holes and smooth transitions or fillets are hence important to increase the fatigue strength of the structure[1].

Fatigue fracture involves following steps:

- Nucleation of crack
- Crack growth
- Crack growing to a tiny size
- Fracture

Nucleation is due to brittle particles, discontinuities, stress raisers like flaws sharp corners, holes, keyways, fillets etc. It is caused when the maximum stress within elastic range fluctuates from tensile to compressive. With continued load cycles, crack grows to microscopic

size and gradually become visible to the eyes. Fracture then occurs when crack reaches to some critical size.

Characteristics of fatigue[1]:

- In metals and alloys, when there are no macroscopic or microscopic discontinuities, the process starts with dislocation movements, eventually forming persistent slip bands that nucleate short cracks.
- Macroscopic and microscopic discontinuities as well as component design features which cause stress concentration (keyways, sharp changes of direction etc.) are the preferred location for starting the fatigue process.
- Fatigue is a stochastic process, often showing considerable scatter even in controlled environments.
- The greater the applied stress range, the shorter the life.
- Fatigue life scatter tends to increase for longer fatigue lives.
- Damage is cumulative. Materials do not recover when rested.
- Fatigue life is influenced by a variety of factors, such as temperature, surface finish, microstructure, presence of oxidizing or inert chemicals, residual stresses, contact (fretting), etc.
- Some materials (e.g., some steel and titanium alloys) exhibit a theoretical fatigue limit below which continued loading does not lead to structural failure.

- In recent years, researchers have found that failures occur below the theoretical fatigue limit at very high fatigue lives (10^9 to 10^{10} cycles). An ultrasonic resonance technique is used in these experiments with frequencies around 10–20 kHz.
- High cycle fatigue strength (about 10^3 to 10^8 cycles) can be described by stress-based parameters. A load-controlled servo-hydraulic test rig is commonly used in these tests, with frequencies of around 20–50 Hz. Other sorts of machines—like resonant magnetic machines—can also be used, achieving frequencies up to 250 Hz.
- Low cycle fatigue (typically less than 10^3 cycles) is associated with widespread plasticity in metals; thus, a strain-based parameter should be used for fatigue life prediction in metals and alloys. Testing is conducted with constant strain amplitudes typically at 0.01–5 Hz.

ASTM defines *fatigue life* as the number of stress cycles of a specified character that a specimen sustains before failure of a specified nature occurs[1].

Factors that affect fatigue-life[1]:

- **Geometry:** Notches and variation in cross section throughout a part lead to stress concentrations where fatigue cracks initiate.
- **Surface quality:** Surface roughness cause microscopic stress concentrations that lower the fatigue strength. Compressive residual stresses can be introduced in the surface by e.g. shot peening to increase fatigue life. Such techniques for producing surface stress are often referred to as *peening*, whatever the mechanism used to produce the stress. Low

plasticity burnishing, laser peening, and ultrasonic impact treatment can also produce this surface compressive stress and can increase the fatigue life of the component. This improvement is normally observed only for high-cycle fatigue.

- **Material Type:** Fatigue life, as well as the behavior during cyclic loading, varies widely for different materials, e.g. composites and polymers differ markedly from metals.
- **Residual stresses:** Welding, cutting, casting, and other manufacturing processes involving heat or deformation can produce high levels of tensile residual stress, which decreases the fatigue strength.
- **Size and distribution of internal defects:** Casting defects such as gas porosity, non-metallic inclusions and shrinkage voids can significantly reduce fatigue strength.
- **Direction of loading:** For non-isotropic materials, fatigue strength depends on the direction of the principal stress.
- **Grain size:** For most metals, smaller grains yield longer fatigue lives, however, the presence of surface defects or scratches will have a greater influence than in a coarse grained alloy.
- **Environment:** Environmental conditions can cause erosion, corrosion, or gas-phase embrittlement, which all affect fatigue life. Corrosion fatigue is a problem encountered in many aggressive environments.
- **Temperature:** Extreme high or low temperatures can decrease fatigue strength.

1.2 NEED FOR MODELLING FATIGUE CRACK GROWTH

Prediction of life of a structural or any machine component is often a challenging procedure in the field of engineering . Because of high cost of critical components, they should be used for optimum life and should be replaced only before a shutdown or failure is due. Also it is important to know the life of a critical component so that it can be replaced before a catastrophic failure occurs. Although various advances have been made in the field of engineering, failure of the components due to fatigue is a common occurrence and determination of the fatigue life of a component is still in its developing stage. It is known that every structures contain flaws in them, which may be due to either from metallurgical defects or from any damage taken during their service life. These cracks and the flaws often grow from an initial size to a critical size, which may lead to the catastrophic failure of the components when they are subjected to the fatigue loading. Design of present day sophisticated equipment, to protect them against fatigue failure is important for both safety and economic point of view. Hence, there is the need for a life prediction methodology so that a component may be replaced before a catastrophic failure occurs. With the application of fracture mechanics concepts to fatigue failure and the development of sophisticated crack detecting techniques, the crack propagation result can be acquired in terms of number of cycles (N) and the crack length (a) .

1.3 THE EXPONENTIAL MODEL

For predicting fatigue life, the rate at which a fatigue crack grows must be suitably described in terms of various crack driving parameters. The aim of developing a fatigue crack growth model is to predict a safe operating life while designing a structure/component subjected to any cyclic loading. Integrating the rate equation of the Paris type may give the desired result of calculating the service life of a structure/machine component under cyclic loading. However, doing the direct integration becomes very complicated as the geometrical factor ' $f(g)$ ' in the expression of ΔK varies with crack length. Therefore, fatigue life may be found by the numerical integration using different values of ' $f(g)$ ' kept constant through a small crack length increment. To overcome this difficulty, an attempt has been made in the present paper to introduce a life prediction methodology by adopting an 'Exponential Model'[2]. It may be noted that the Exponential model is quite often used for calculation of growth of population/bacteria etc. This concept has been made use in the present context to develop fatigue crack growth model. The model can predict the fundamental a - N curve to calculate life without integration of FCGR curve. To calculate the specific crack growth rate in the exponential model, dimensional analysis has been used.

CHAPTER 2

LITERATURE REVIEW

Several attempts have been made till now by various researchers to find the fatigue crack growth rate using either by graphical procedures or by computational methods and to establish some functional relationship between fatigue crack growth rate and different variables. With the introduction of fracture mechanics and its use in characterizing fatigue crack growth, many important work has been done in this area, particularly the work of Paris et.al. [3,4]. The most notable and basic model proposed by Paris in 1960 is:

$$da/dN = C(\Delta K)^n$$

where C and n are material constants, and ΔK is the stress intensity factor range given by $K_{\max} - K_{\min}$.

The above relation is valid for the Region II of the fatigue crack propagation.

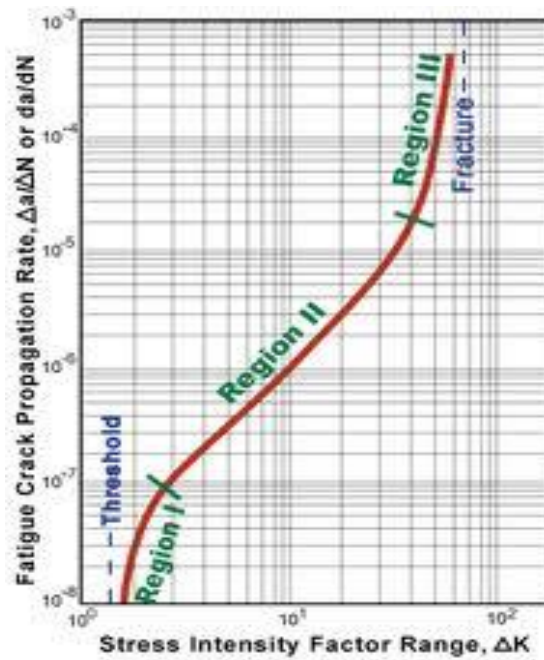


Fig.1. Fatigue crack growth rate vs stress intensity factor range [5]

In Region-III, unstable crack growth occurs due to the occurrence of static fracture modes such as inter-granular fracture, cleavage etc. In many practical engineering situations this region is ignored because it does not significantly affect the total crack propagation life. The crack propagation transition from Region-II to Region-III is dependent on the yield strength of the material.

Since then many models have been proposed to predict fatigue crack growth rate and fatigue life. Newman et. al [6] studied the fatigue and crack growth in 7050-T7451 aluminum alloy under constant-and variable-amplitude loading. Bahashwan, Abdulla[7] proposed their paper on determination of fatigue crack growth rate of single tensile peak loads from constant amplitude loading. Tokaji[8] studied the small fatigue crack growth behaviour under two stress level multiple loading. Tadjiev [9] did fatigue crack growth prediction in 7475-T7351 aluminium alloy under random loading using modified root mean square model. Alawi[10] proposed a probabilistic model for fatigue crack growth under random loading .The work in recent years include those of Pereira et. al [11]who predicted fatigue crack retardation in 7150 aluminum alloy. Xiong [12] gave new model of two-parameter driving force for fatigue crack growth.

Various efforts have been adopted to correlate and develop relationship between fatigue crack growth and the various loading conditions. Mohanty *et. al* (2009)[2] proposed an exponential model to predict fatigue crack growth in structures under constant amplitude loading. The proposed model considers the change in stress intensity factor which changes with increase in the crack size. The model is based on the equation known as the law of growth. The important parameter of this equation is specific growth rate which depends on various loading conditions. The specific crack growth rate, m is assumed to depend on crack driving forces ΔK and K_{\max} and

material parameters K_c , E , and σ_{ys} . This exponential model is valid for both Region-II and Region-III. The general formula is given by[2] :

$$a_j = a_i e^{m(N_j - N_i)}$$

where a_i and a_j is the crack length in i th step and j th step in mm , respectively; N_i and N_j is number of cycles in i th step and j th step, respectively; m is specific growth rate in the interval $i-j$; i is the number of experimental steps.

The specific crack growth rate from above model is[2]:

$$m = \frac{\ln(\frac{a_j}{a_i})}{(N_j - N_i)}$$

The fatigue life according to above exponential model is[2]:

$$N_j = N_i + \frac{\ln(\frac{a_j}{a_i})}{m}$$

CHAPTER 3

EXPERIMENTAL DETAILS

3.1 Experimental procedure:

The experimental data used in the present project has been obtained from the experiments performed by Mohanty J R[2].The experiment was done using 7020 and 2024 Al-alloys. The 7020 Al-alloy, which is suitable for ground transport system was procured in the as-fabricated condition, while 2024 Al-alloy was procured in T3 heat-treated condition. The 7020 Al-alloy was subjected to T7 heat-treatment to obtain desired mechanical properties, as required according to the experiment. The specimen used in this experiment was Single-edge notched specimens. Specimens with a thickness of 6.5 mm were used for conducting the fatigue test. The specimens were made in the LT plane, and the loading was aligned in the longitudinal direction. To perform the experiments an Instron-8502 machine was used with 250 kN load cell capacity. The machine was interfaced to a computer for control and data acquisition. The experiment was conducted in air and at room temperature. For this experiment fatigue pre-cracked specimen under mode-I loading was used with an a/w ratio of 0.3 and these specimens were subjected to constant load test (i.e. progressive increase in ΔK with crack extension) with a load ratio of 0.1. The sinusoidal load cycles were applied to the specimen at a frequency of 6 Hz. To monitor the crack growth a COD gauge was used on the face of the notch of the specimen. The stress intensity factor , K, is calculated based on the following equation [13]

$$K = f(g).(F\sqrt{\pi * a})/(w.B)$$

Where , $f(g) = 1.12 - 0.231(a/w) + 10.55(a/w)^2 - 21.72(a/w)^3 + 30.39(a/w)^4$



Fig.2. Instron 8502 with SENT specimen

3.2 Specimen details:

For the fatigue crack growth tests performed by Mohanty J R[2], single edge notch tension (SENT) specimens were fabricated from the given 6.5mm thick sheet. These specimens were made in the LT plane, and the loading was aligned in the longitudinal direction. The details of dimension of specimen are shown in Fig.

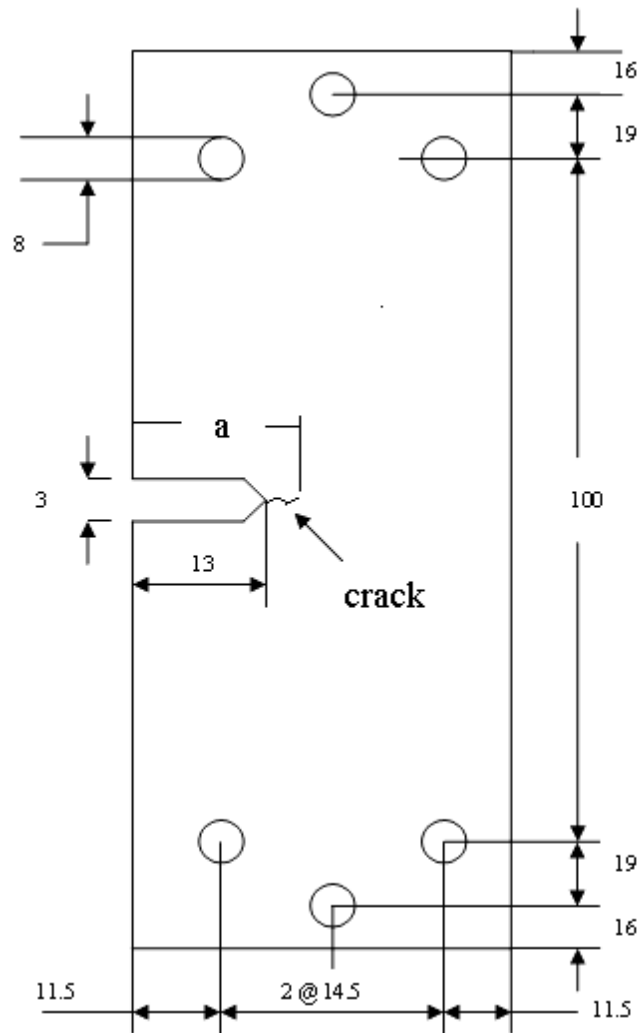


Fig. 3 – Single Edge Notch (SEN) Specimen geometry (LT orientation)

Table 1 – Chemical composition of materials

Materials (% wt.)	Al	Cu	Mg	Mn	Fe	Si	Zn	Cr	Others
Al 7020	93.13	0.05	1.20	0.43	0.37	0.22	4.60	-	-
Al 2024	92.78	3.90	1.50	0.32	0.50	0.50	0.25	0.10	0.15

Table 2 – Mechanical properties of materials

Materials	Tensile strength (σ_{ut}) MPa	Yield strength (σ_{ys}) MPa	Young's modulus (E) MPa	Poisson's ratio	Plane- Strain Fracture toughness (K_{IC}) MPa \sqrt{m}	Plane- Stress Fracture toughness (K_C) MPa \sqrt{m}	Elongation
Al 7020	352.14	314.70	70,000	0.33	50.12	236.80	21.54 % in 40 mm
Al 2024	469.00	324.00	73,100	0.33	37.00	95.31	19% in 12.7 mm

CHAPTER 4

MODEL FORMULATION

3.1 Formulation of exponential model

The following exponential model has been developed by Mohanty et al.[2].For the calculation of growth of population/bacteria following exponential model is generally used. The differential equation representing the growth is given as[2]

$$\frac{dP}{dt} = rP$$

Where t is the time and P, population.

The solution of this differential equation is given as

$$P(t) = P_o.e^{rt}$$

Where r is the specific growth rate

The above model has been utilized to formulate exponential model for fatigue crack growth, which is given as[2]

$$a_j = a_i e^{m(N_j - N_i)}$$

where m is the specific crack growth rate in the interval i-j and is given as[2]

$$m = \frac{\ln(\frac{a_j}{a_i})}{N_j - N_i}$$

where a_i and a_j is the crack length in ith step and jth step in ‘mm’, respectively; N_i and N_j is number of cycles in ith step and jth step, respectively.

It has been found that the fatigue crack growth behavior depends on the initial crack length and the previous load history ,hence while using the above model, each previous crack length is taken as the initial crack length for the present step.

3.2 Calculation of specific crack growth rate

To calculate specific growth rate m , for use in the above exponential model, approach of dimensional analysis has been used ,to develop expression of m in terms of different depending parameters.

Specific crack growth rate is an important parameter which depends on crack driving forces and material properties. As per the ‘Unified Approach’, fatigue crack growth rate depends on the crack driving force ΔK , as well as on K_{\max} so that the mean stress effects could be taken into account. Hence, m also depends on ΔK and K_{\max} [14]. m has also been found to depend on plane stress fracture toughness K_c of the material because the present modeling covers region III of fatigue crack growth rate curve[2]. It also depends on initial crack length a , because in constant load fatigue test, crack length increases with number of cycles resulting in increase in stress intensity factor, thus m also changes with crack length. Similarly m depends on load ratio R and σ_y . The dependence of m on these parameters is given by following equations[15]

$$m = \Phi(\Delta K, K_c, K_{\max}, \sigma_y, 1/a, R)$$

using dimensional analysis , above relation can be represented as

$$f(m, \Delta K, K_c, K_{\max}, \sigma_y, 1/a) = 0$$

$$\Phi(\pi_1, \pi_2, \pi_3) = 0$$

Let the repeating variables are ΔK , K_c and $1/a$

$$\pi_1 = (\Delta K)^{a1} (K_c)^{b1} (1/a)^{c1} . m$$

Substituting the dimensions of each term in above equation, and solving for $a1, b1, c1$, we get

$$\pi_1 = m . (K_c / \Delta K)^{a1}$$

$$\text{Again , } \pi_2 = (\Delta K)^{a2} (K_c)^{b2} (1/a)^{c2} . K_{\max}$$

Substituting the dimensions of each term in above equation, and solving for $a2, b2, c2$, we get

$$\pi_2 = \frac{K_{\max}}{K_c} . (\Delta K / K_c)^{a2}$$

$$\text{Again , } \pi_3 = (\Delta K)^{a3} (K_c)^{b3} (1/a)^{c3} . \sigma_y$$

Substituting the dimensions of each term in above equation, and solving for $a3, b3, c3$, we get

$$\pi_3 = (\Delta K / K_c)^{a3} . \frac{\sigma_y}{K_c} . \sqrt{a}$$

Since any π term can be represented as the function of other π terms, we have

$$\pi_1 = \Phi(\pi_2, \pi_3)$$

$$m . (K_c / \Delta K)^{a1} = \Phi \left[\frac{K_{\max}}{K_c} . (\Delta K / K_c)^{a2}, (\Delta K / K_c)^{a3} . \frac{\sigma_y}{K_c} . \sqrt{a} \right]$$

$$m = \left(\frac{\Delta K}{K_c} \right)^{a1} . \Phi \left[\frac{K_{\max}}{K_c} . (\Delta K / K_c)^{a2}, (\Delta K / K_c)^{a3} . \frac{\sigma_y}{K_c} . \sqrt{a} \right]$$

$$m = \left(\frac{\Delta K}{K_c} \right)^{a1} . \left(\frac{K_{\max}}{K_c} \right)^{a2} . (\sigma_y^2 . a / K_c^2)^{a3}$$

If we consider the effect of load ratio also, then final expression for specific crack growth rate can be written as

$$m = \left(\frac{\Delta K}{K_c}\right)^{a_1} \cdot \left(\frac{K_{\max}}{K_c}\right)^{a_2} \cdot (\sigma_y^2 \cdot a / K_c^2)^{a_3} \cdot (1-R)^{a_4}$$

where a_1, a_2, a_3 and a_4 are the constants whose values are to be determined. This is done using MATLAB software, where experimental data are used to calculate the values of the constants.

Finally the expression of specific crack growth rate is obtained as following

$$m = (\Delta K / K_c)^{45.6738} (K_{\max} / K_c)^{-43.1861} (\sigma_y^2 \cdot a / K_c^2)^{1.038} (1-R)^{-1.0943}$$

CHAPTER 5

RESULTS AND DISCUSSIONS

5.1 Experimental results:

Fatigue test was performed on the SENT specimen by Mohanty J R[2] , and following data and results were obtained from the experiment:

Table 3 Experimental data for a and N (at $R = 0$) and calculated values of m [2]

N	a	m
70500	16.4	
72233.01	16.45	1.75656E-06
73883.54	16.5	1.83874E-06
75456.85	16.55	1.92316E-06
76957.85	16.6	2.00973E-06
78391.1	16.65	2.09839E-06
79760.86	16.7	2.18907E-06
81071.08	16.75	2.2817E-06
82325.44	16.8	2.37622E-06
83527.35	16.85	2.47255E-06
84679.97	16.9	2.57063E-06
85786.25	16.95	2.67041E-06
86848.92	17	2.7718E-06
87870.52	17.05	2.87475E-06
88853.42	17.1	2.97921E-06
89799.81	17.15	3.0851E-06
90711.74	17.2	3.19237E-06
91591.1	17.25	3.30096E-06
92439.69	17.3	3.41081E-06
93259.14	17.35	3.52187E-06
94051.01	17.4	3.63408E-06
94816.73	17.45	3.74738E-06
95557.65	17.5	3.86172E-06
96275.03	17.55	3.97705E-06
96970.06	17.6	4.09331E-06

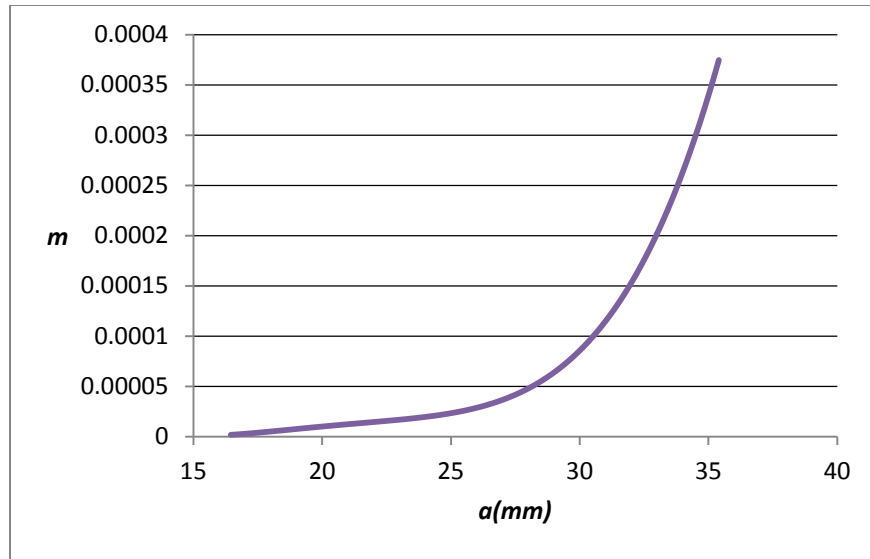


Fig.4 m vs a curve for Al 2024 at $R=0$

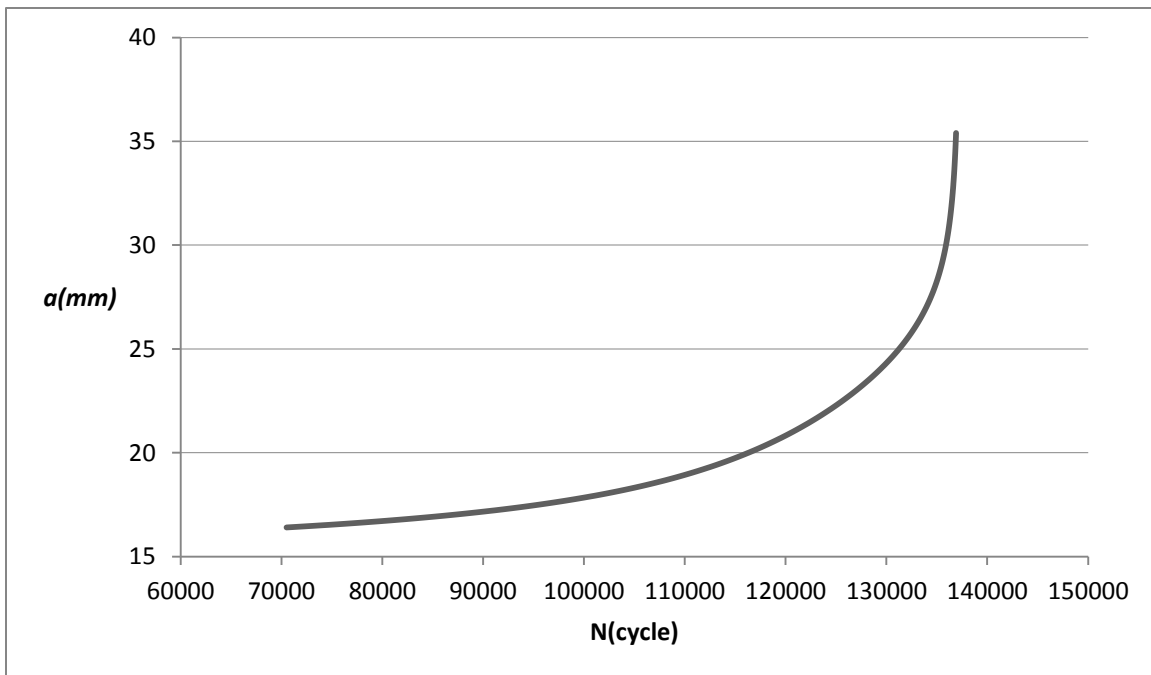


Fig. 5 a vs N curve for Al 2024 at $R=0$

5.2 Comparison of predicted and experimental results (Validation of proposed exponential model):

The present model has been tested by comparing the result obtained from experimental data with the predicted results for load ratio of $R= 0.2$ for Al 2024 specimens.

Table 4 Experimental and Predicted result comparison

Experimental data for R=0.2			Predicted result from model		
N	a	m	m	N	a
70500	16.4				
71857.17	16.45	2.24E-06	4.3E-06	71207.45	16.49605
73128.92	16.5	2.39E-06	4.36E-06	72553.93	16.54138
74324.84	16.55	2.53E-06	4.41E-06	73815.18	16.58723
75453	16.6	2.67E-06	4.46E-06	75000.76	16.63354
76520.26	16.65	2.82E-06	4.52E-06	76118.75	16.68023
77532.51	16.7	2.96E-06	4.57E-06	77176	16.72725
78494.84	16.75	3.11E-06	4.63E-06	78178.41	16.77455
79411.69	16.8	3.25E-06	4.68E-06	79131.06	16.8221
80286.92	16.85	3.4E-06	4.74E-06	80038.38	16.86987
81123.95	16.9	3.54E-06	4.8E-06	80904.24	16.91783
81925.79	16.95	3.68E-06	4.86E-06	81732.05	16.96596

Following figures show the experimental as well as predicted a -N curve for the tested specimens. The experimental and the predicted results of (da/dN) vs ΔK plots have also been shown. It is observed that experimental and predicted results match with each other.

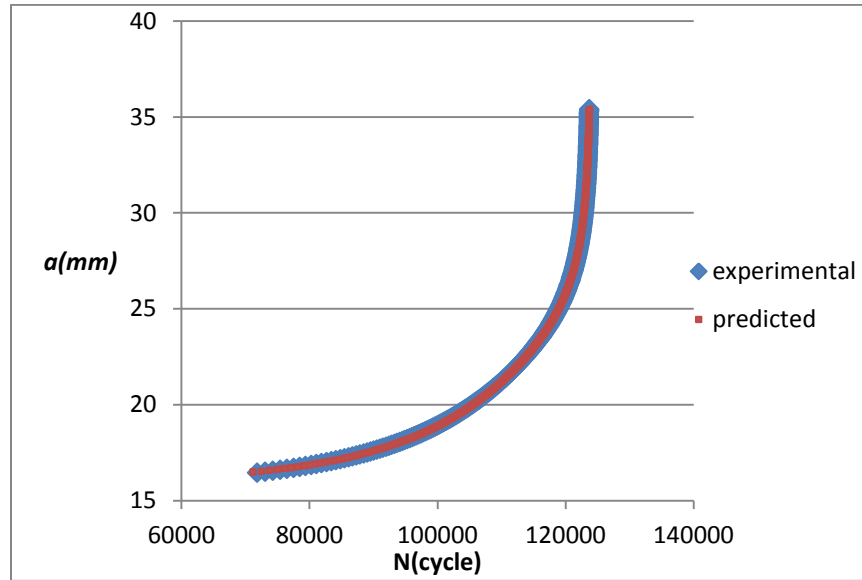


Fig. 6 Experimental and predicted a - N curve for Al 2024 at $R=0.2$

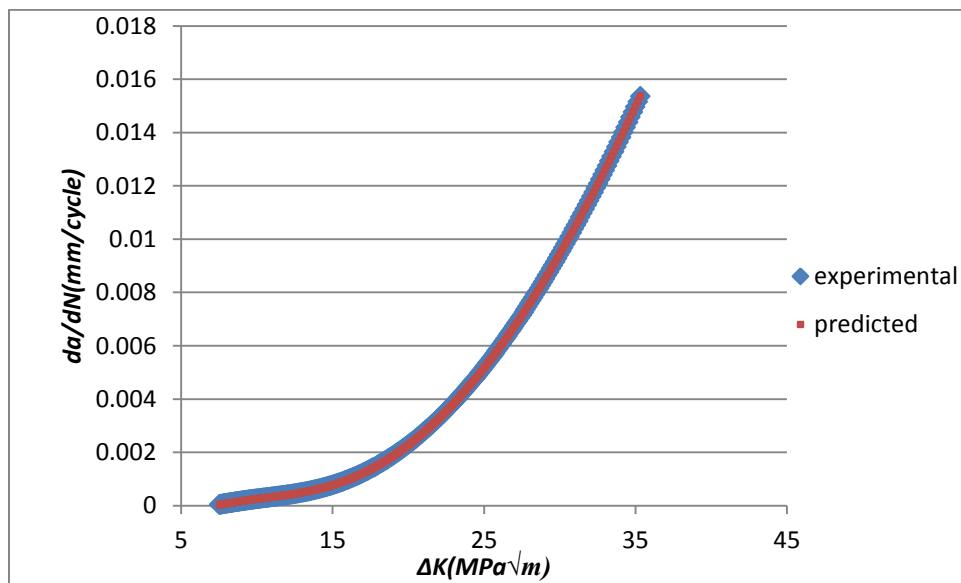


Fig.7 Experimental and predicted da/dN - ΔK curve for Al 2024 at $R=0.2$

CHAPTER 6

CONCLUSION

Following conclusions can be made:

- Exponential model of the form $a_j = a_i e^{m(N_j - N_i)}$, can be used effectively to find fatigue crack growth and residual life in constant amplitude loading[2]. Here m is the specific crack growth rate in the interval $i-j$, a_i and a_j is the crack length in i th step and j th step in mm , respectively; N_i and N_j is number of cycles in i th step and j th step, respectively.
- The important parameter in the proposed exponential model is the Specific crack growth rate, which depends on different parameters involving crack driving forces and material properties like ΔK , K_c , K_{max} , σ_y , and crack length a . Hence it can be represented using dimensional analysis by the expression

$$m = (\Delta K/K_c)^{a1} (K_{max}/K_c)^{a2} (\sigma_y^2 \cdot a/K_c^2)^{a3} (1-R)^{a4}$$

$$m = (\Delta K/K_c)^{45.6738} (K_{max}/K_c)^{-43.1861} (\sigma_y^2 \cdot a/K_c^2)^{1.038} (1-R)^{-1.0943}$$

- The results from predicted model has been found to be in good agreement with the experimental results obtained from Al 2024 alloy for load ratio, $R=0.2$.

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Appendix

1.Sample calculation

Table 5

<p>For the specimen Al 2024 , we have $\sigma_y = 324 \text{ MPa}$ $K_c = \text{plane stress fracture toughness} = 95.31 \text{ MPa}\sqrt{m}$ $w = \text{width of specimen} = 52 \text{ mm}$ $B = \text{thickness of specimen} = 6.5 \text{ mm}$ $K = \text{stress intensity factor} = f(g).(F\sqrt{\pi * a})/(w.B)$ Where , $f(g) = 1.12 - 0.231(a/w) + 10.55(a/w)^2 - 21.72(a/w)^3 + 30.39(a/w)^4$</p>					
a	m	$f(g)$	K_{\max}	K_{\min}	ΔK
16.4		1.715832	8.294255	0.829425	7.464829
16.45	2.24E-06	1.719451	8.324409	0.832441	7.491968
16.5	2.39E-06	1.723085	8.35467	0.835467	7.519203
16.55	2.53E-06	1.726734	8.385039	0.838504	7.546535
16.6	2.67E-06	1.730399	8.415517	0.841552	7.573966
16.65	2.82E-06	1.734078	8.446105	0.844611	7.601495
16.7	2.96E-06	1.737774	8.476804	0.84768	7.629123
16.75	3.11E-06	1.741485	8.507613	0.850761	7.656852
16.8	3.25E-06	1.745211	8.538534	0.853853	7.684681
16.85	3.4E-06	1.748954	8.569568	0.856957	7.712611

(The data used here are obtained from experiments performed by Mohanty JR[2].)

2.Dimensions for different variables for dimensional analysis:(Table 6)

Variable	Symbol	Dimension
Yield tensile strength	σ_y	$FL^{-2}T^0$
Fracture toughness	K_c	$FL^{-3/2}T^0$
Crack length	a	F^0LT^0
Stress intensity factor range	ΔK	$FL^{-3/2}T^0$
Specific crack growth rate	m	$F^0L^0T^0$